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IV. RECENT RESULTS OF MEASUREMENTS

Plume rise estimates for electric generating stations

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The Tennessee Valley Authority, under sponsorship of the Public Health Service, National Air Pollution Control Administration, initiated a comprehensive study titled 'Full scale study of plume rise at large electric generating stations' in 1963. The variability of plant sizes, stack heights, and stack configurations accommodated full scale assessment of plume rise over a wide range of meteorological and operational conditions.

Introduction

There are three principal requirements for determining the potential ground level pollution from large industrial stack emissions such as those at electric generating stations. These factors are: (1) an estimate of the rate of emission from the stack (emission rate) which is normally available from operations information, (2) the degree of dilution of the pollutant with the surrounding atmosphere (rate of dispersion), and (3) the sum of actual stack height and vertical displacement of the plume (effective stack height) which is influenced by the diameter of the stack nozzle, the temperature of effluent gases with respect to surrounding air, the specific heat capacity at constant pressure, and the average vertical efflux velocity.

The excellent work of Sutton (1953) and Pasquill (1962) in England, along with investigations in the U.S., has led to the development and verification of mathematical formulas which are considered suitable to estimate diffusion rates for principal meteorological dispersion models. Diffusion is generally represented as a 2-phase problem—the first is the initial rise of the efflux by virtue of its kinetic and thermal energy and the second when it begins ist spread downwind from the effective source height it has attained.

As pointed out at the Round Table Discussion of dispersion, convened by Concawe at the Hague, November 1967, much ambiguity exists with respect to the numerous formulas postulated to approximate effective plume height for defined meteorological and operational conditions. The elevation of effective stack height, H_e, is the sum of the actual height of the stack, h_s, and of the plume rise, Δh. The equations used in this paper for calculation of Δh may be summarized as

$$\Delta h = A/u^a, \tag{1}$$

where A is some function of the kinetic and thermal energy of the plume and u is the wind speed.

Basic workplan

The objective of this study was to collect, compile, and analyse data for documentation and definition of plume rise and related meteorological parameters at a range of generating plants. Six generating plants were chosen for the study with unit ratings from 173 to 704 MW and stack heights from 76.2 to 182.9 m.

The first two years of the study, completed in the spring of 1965, were devoted primarily to collection of field plume rise and meteorological data. Field work, totalling about 311 h of actual sampling, was scheduled in seasons when frequencies of desired meteorological régimes were

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expected to be the highest, i.e. high winds and neutral stability in the spring (March to April) and low winds and stable or inversion conditions in the fall (September to October). A third régime, i.e. low winds and unstable conditions, was also documented whenever possible. Procedural aspects of the study were presented in an interim report (Leavitt, Carpenter & Thomas 1965).

Data compilations

A comprehensive data collection programme was devised. The three general categories of data included: (1) plant design and operational factors, (2) meteorological information, and (3) plume profile, elevation of plume top and bottom.

Plant design and operational factors

Principal plant design and operational factors for the six steam plants are shown in table 1.

For each field study period the amount, power rating, and approximate analysis of coal burned for each unit and the unit level of operation were obtained for determination of stack effluent velocity and heat emission rate. Gas temperatures at the stack outlet were also obtained from a sampling programme developed especially for the study.

Table 1. Plant design and operational data

	Paradise	Gallatin	Shawnee	Johnsonville	Colbert	Widows Creek
plant number	1	2	3	4	5	6
number of units	2	4	10	4	4	1
rated capacity, per unit/MW	704	314	175	172.8	206	575
stacks: number	2	2	10	2	4	1
height/m	182.9	152.4	76.2	121.9	91.4	152.4
diameter/m	7.9	7.6	4.3	4.3	5.0	6.3
spacing/m	61.9	77.4	25.2	49.4	30.2	—
temperature of flue gas leaving stack/K	413.2	409.9	413.2	424.9	443.8	414.3
orientation	N 38° E	N 39° E	N 56° W	N 5° W	S 45° E	—

Meteorological information

Wind direction, wind speed, and temperature profile data were collected routinely. Wind profile data were obtained at approximately 30 min intervals by the double theodolite technique.

Vertical temperature profiles were obtained from a Bell Model 47-D-1 helicopter equipped with a Cole–Parmer model 8425 temperature indicator. Temperature profile runs were made at 45 min intervals in the immediate plume area about 1.6 km from the power plant. The 305 individual temperature profiles were taken at 30.5 m vertical intervals from surface to about 150 m above the plume top and involved a total helicopter time of 188 h.

Additional meteorological information, i.e. surface wind direction, wind speed, and dry-bulb and wet-bulb temperatures, was recorded before each pibal release at the primary station. Also, cloud coverage and other pertinent meteorological or plume observational information was recorded.

General synoptic weather information from U.S. Weather Bureau radiosonde observations and surface and upper air charts was also compiled for analysis and evaluation.

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Range of plant operational and meteorological conditions covered

The range of principal plant operational and meteorological conditions spanned by these observations is summarized as follows:

stack gas velocity	7.7 to 29.2 $\mathrm{m\,s^{-1}}$,
volume emission rate	136 to 663 $\mathrm{m^3\,s^{-1}}$,
stack gas temperature	379 to 418 K (106 to 145 °C),
heat emission rate	22.1 to 103 $\mathrm{MJ\,s^{-1}}$ (5.28 to 24.6 $\mathrm{Mcal\,s^{-1}}$),
wind speed, $\bar{u}$	1.0 to 16.8 $\mathrm{m\,s^{-1}}$,
ambient temperature	273 to 304 K (0 to 31 °C),
potential temperature gradient	-5.3 to 37.4 $\mathrm{K\,km^{-1}}$.

Plume profile

In the course of the study, 1580 photographs were taken for definition of the plume profile. In addition to excellent quality control, infrared film was found to provide superior plume delineation in most cases and was used in preference to standard black and white or Polaroid film. The photographs were taken at 5 min intervals.

Elevation of top and bottom of the plume was also recorded about every 45 min at 0.8 and 1.6 km, and at 3.2 km when possible, by visual observation and helicopter altimeter readings. The helicopter was also used during each study period to maintain continuous surveillance on the direction of plume travel. These observations were obtained by references to established ground control points.

Data processing and tabulation

All data from photographs, pilot balloon observations, and temperature soundings were programmed for computer analysis and graphic display. Plume profile data from photographs were resolved by means of special template overlays constructed to coincide with the particular camera lens. Elevation of plume top and bottom observed from the helicopter agreed closely with corresponding points on the plume profile obtained from the photographs.

For each series of photographs the mean height of top and bottom of the plume was determined as a function of distance from the plant; and from these values a plot of the centreline position was derived, the centreline being defined as the arithmetic mean between the top and the bottom. From these plots, rise of the centreline as a function of distance was determined. Figure 1 illustrates a portion of the 1 April 1965 field data, including the plume photograph taken at 07.15, along with concurrent wind and temperature profiles and computer resolved plume profile plots.

The 1580 separate plume rise observations taken at 5 min intervals at the six plants were consolidated into 133 composite observations, each covering time periods of 30 to 120 min. Duration of composite periods was determined by the constancy of principal meteorological and operational parameters. Therefore each of the composite observations represents from six to twenty-four 5 min consecutive observations. The composite observations were next classified into stability groups based on the temperature gradient from stack top to plume top. The stability classifications and ranges were:

stability classification	temperature gradient range K/100 m	group
inversion	$\Delta\theta/\Delta z > 1.00$	1
stable	$0 < \Delta\theta/\Delta z \leqslant 1.00$	2
neutral and unstable	$\Delta\theta/\Delta z \leqslant 0$	3

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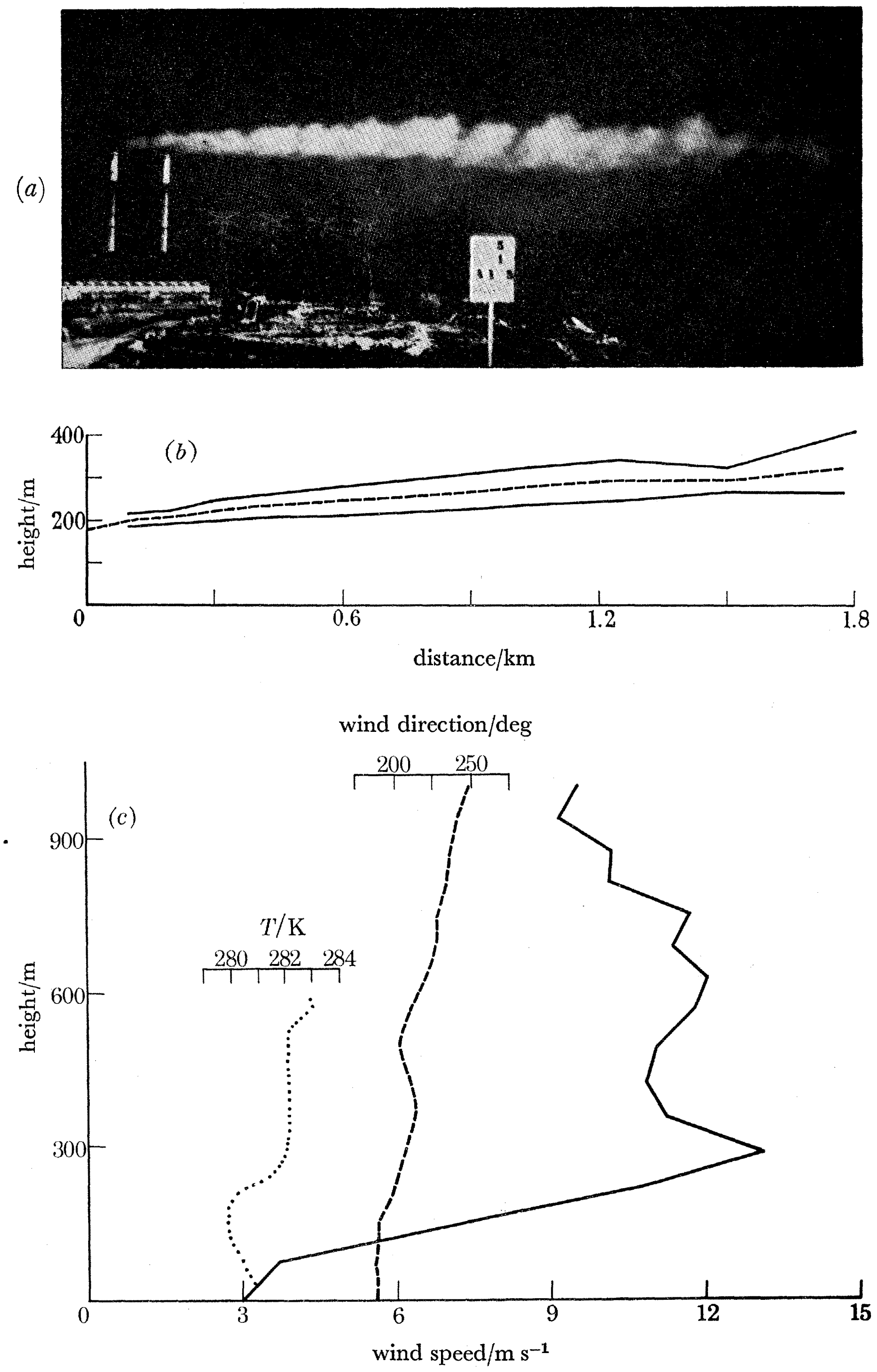

FIGURE 1. Data from typical day of field operation at the Paradise steam plant on 1 April 1965. (*a*) Plume photograph at 07.15 h; (*b*) wind speed and direction profile at 07.16 h, and temperature profile at 07.20 h; (*c*) plume profile derived from photographs: the stack height is 183 m.

The sixty-six composite observations for single stack operation classified according to these stability groups and average wind speed classes are compiled in table 2. Also included in this table are heat emission rate, flue gas velocity, and average plume rise at nine specified downwind distances out to 1.219 km.

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Table 2. Average plume rise with distance from source by stability classes single stack operation

steam plant number	number of observations	$10^{-7}\,Q_H$ / cal s^{-1}	temperature gradient $\frac{\Delta\theta}{\Delta z} / \frac{\mathrm{K}}{100\ \mathrm{m}}$	stability group†	V_s/m s^{-1}	$\bar{u}$/m s^{-1}	plume rise (m) at distance downwind (m)									
							0	76	152	305	457	610	762	914	1067	1219
1	1	1.70	1.73	1	15.6	8.5	0	20	32	46	44	49	62	74	86	89
	4	1.86	1.37	1	16.4	8.1	0	31	44	66	84	96	100	98	104	115
	average	1.78	1.55		16.0	8.3	0	26	38	56	64	73	81	86	95	102
	38	2.44	1.12	1	19.1	16.8	0	16	30	48	62	77	91	104	114	124
2	21	1.77	10.6	1	15.3	4.6	25	46	67	109	142	156	163	166	168	171
6	4	1.79	1.32	1	22.9	2.5	72	190	262	323	356	345	377	—	—	—
1	10	2.02	0.01	2	17.8	1.9	14	118	168	261	302	307	331	323	330	307
	9	2.02	0.17	2	17.8	3.3	12	92	145	203	217	250	283	313	338	363
	13	1.87	0.30	2	17.1	3.4	94	178	240	312	358	394	417	433	447	457
	12	1.87	0.65	2	17.1	3.7	12	80	124	175	214	247	277	307	334	363
	average	1.93	0.28		17.5	3.1	33	117	169	238	273	300	327	344	362	373
	8	2.02	0.44	2	17.8	4.1	24	48	77	126	166	203	230	255	275	295
	11	1.87	0.59	2	17.1	4.5	3	66	107	169	214	247	273	292	309	329
	7	1.86	0.01	2	16.4	4.7	0	57	104	160	185	191	195	193	183	187
	30	2.00	0.08	2	16.4	5.2	0	44	74	119	166	194	227	260	289	308
	43	2.46	0.11	2	19.2	5.2	0	53	87	121	137	141	165	182	—	—
	average	2.04	0.25		17.4	4.7	6	54	90	139	174	195	218	236	264	280
	5	1.86	0.85	2	16.4	6.0	2	59	80	112	136	153	165	173	182	188
	22	1.84	0.28	2	15.6	7.7	0	33	62	92	115	140	159	182	201	219
	average	1.85	0.57		16.0	6.9	1	46	71	102	126	147	162	178	192	204
	39	2.44	0.37	2	19.1	13.8	0	18	34	55	74	95	112	125	139	150
	42	2.46	0.27	2	19.2	8.0	0	37	62	88	106	133	154	175	191	—
	41	2.44	0.09	2	19.1	8.4	0	27	40	51	66	86	113	131	143	154
	average	2.45	0.18		19.2	8.2	0	32	51	70	86	110	134	153	167	154
	28	2.18	0.02	2	18.0	10.0	0	34	50	61	64	77	86	94	—	—
	40	2.44	0.02	2	19.1	10.6	0	14	26	52	84	102	116	134	158	165
	26	2.18	0.01	2	18.0	10.8	0	26	43	63	78	93	110	106	—	—
	25	1.84	0.18	2	15.6	11.4	0	13	26	47	62	70	85	93	105	135
	average	2.16	0.06		17.7	10.7	0	22	36	56	72	86	99	107	132	150

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TABLE 2 (*cont.*)

steam plant number	number of observations	$10^{-7} Q_H$ / cal s^{-1}	temperature gradient $\frac{\Delta\theta}{\Delta z} / \frac{\mathrm{K}}{100\ \mathrm{m}}$	stability group†	V_s/m s^{-1}	$\bar{u}$/m s^{-1}	plume rise (m) at distance downwind (m)									
							0	76	152	305	457	610	762	914	1067	1219
2	15	1.70	0.30	2	15.3	2.1	476	451	430	—	—	—	—	—	—	—
	13	1.70	0.09	2	15.3	2.2	56	124	192	322	382	414	444	417	460	450
	4	1.76	0.41	2	15.4	2.5	0	22	43	87	130	173	216	248	276	303
	23	1.77	0.36	2	15.3	2.5	73	112	150	228	300	341	347	322	287	270
	3	1.76	0.63	2	15.4	2.8	0	20	41	81	121	161	202	239	276	312
	17	1.71	0.44	2	15.6	2.9	32	70	109	185	238	248	259	266	317	300
	28	1.52	0.06	2	15.2	3.1	1	47	63	83	112	135	166	196	220	250
	14	1.70	0.23	2	15.3	2.1	27	72	117	171	224	285	335	386	464	555
	22	1.77	0.97	2	15.3	3.2	66	95	124	182	234	249	251	243	235	228
	average	1.71	0.39		15.3	2.6	104	113	141	167	218	251	278	290	317	334
	25	1.50	0.10	2	14.8	4.6	0	43	66	90	128	171	192	227	247	259
	7	1.74	0.22	2	16.1	5.0	0	16	32	63	95	127	159	178	192	207
	19	1.65	0.68	2	15.5	5.0	19	43	68	118	161	168	181	194	210	220
	16	1.71	0.74	2	15.6	5.1	21	49	76	132	175	184	194	208	215	226
	24	1.50	0.61	2	14.8	5.2	5	81	187	183	220	250	274	310	335	359
	9	1.75	0.39	2	16.4	5.4	0	17	34	69	103	138	172	189	198	208
	8	1.74	0.11	2	16.1	5.8	0	16	31	62	94	125	156	183	199	215
	6	1.74	0.72	2	16.1	5.9	0	13	26	52	78	104	130	144	155	165
	average	1.67	0.45		15.7	5.3	6	35	58	96	132	158	182	204	219	232
	10	1.73	0.30	2	15.4	6.9	0	16	31	62	93	124	155	167	173	180
	27	1.50	0.13	2	14.8	10.4	0	26	46	72	91	103	109	143	165	—
6	6	1.79	0.04	2	22.9	2.2	130	258	314	331	333	—	—	—	—	—
	10	1.57	0.09	2	22.9	2.4	22	101	136	205	286	316	278	—	—	—
	5	1.79	0.49	2	22.9	3.4	56	120	163	208	243	304	—	—	—	—
	average	1.72	0.21		22.9	2.7	69	160	204	248	287	310	278	—	—	—
	9	1.57	0.25	2	22.9	4.1	18	81	111	154	190	214	238	301	—	—
	3	1.77	0.02	2	22.9	4.8	5	37	57	80	103	130	155	155	149	—
	8	1.57	0.22	2	22.9	5.2	18	63	86	126	129	134	138	—	—	—
	average	1.64	0.16		22.9	4.7	14	60	85	120	141	159	177	228	149	—

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	1	1.77	0.19	2	22.9	6.5	0	21	42	61	77	93	103	117	117	136
1	6	1.86	0	3	16.4	2.6	1	97	160	228	269	304	327	340	354	375
	3	1.70	−0.09	3	15.6	4.9	0	59	93	109	110	147	184	221	256	—
	31	2.00	−0.02	3	16.4	5.7	0	59	94	117	132	154	152	160	171	178
	average	1.85	−0.06		16.0	6.3	0	59	94	113	121	151	168	191	214	178
	2	1.70	−0.05	3	15.6	6.8	0	31	50	67	71	95	119	143	167	179
	24	1.84	−0.13	3	15.6	7.2	0	16	29	50	76	96	111	122	139	145
	21	1.78	−0.17	3	15.3	7.3	0	34	54	78	102	126	148	167	188	242
	average	1.77	−0.12		15.5	7.1	0	27	44	65	83	106	126	144	165	189
	23	1.84	−0.04	3	15.6	10.9	0	25	44	65	84	112	115	133	154	177
2	12	1.70	−0.03	3	15.3	3.8	13	64	93	127	160	176	214	—	—	—
	29	1.52	−0.00	3	15.2	4.4	0	59	74	111	142	171	198	228	251	263
	average	1.61	−0.02		15.3	4.1	7	62	84	119	151	174	206	228	251	263
	20	1.65	−0.04	3	15.5	5.8	16	46	76	135	194	251	286	321	330	354
	18	1.71	−0.02	3	15.6	6.1	14	37	62	109	157	196	220	251	276	214
	11	1.73	−0.16	3	15.4	6.9	0	9	19	37	55	74	93	111	130	—
	average	1.70	−0.07		15.5	6.3	15	31	52	94	135	174	200	228	245	334
	26	1.50	0	3	14.8	9.9	0	24	45	63	81	99	98	109	109	137
1	29	2.18	−0.02	3	180.	8.3	0	27	41	54	60	69	88	102	109	98
	27	2.18	0	3	18.0	9.5	0	25	43	62	67	89	107	103	108	—
	average	2.18	−0.01		18.0	8.9	0	26	42	58	64	79	98	103	109	98
6	2	1.77	−0.17	3	22.9	6.0	0	23	44	63	77	99	103	125	119	—
	7	1.59	−0.18	3	24.5	6.2	1	25	71	73	—	—	—	—	—	—
	average	1.68	−0.18		23.7	6.1	0	24	58	68	77	99	103	125	119	—

† Stability classification group

1	2	3
$\frac{\Delta\theta}{\Delta z} > 1$	$0 < \frac{\Delta\theta}{\Delta z} \leqslant 1$	$\frac{\Delta\theta}{\Delta z} \leqslant 0$

where $\frac{\Delta\theta}{\Delta z} = \mathrm{K}/100\ \mathrm{m}$.

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DATA ANALYSIS AND EVALUATION

Point of effective plume rise

Often the most difficult point to establish from a plume profile is the height which the rise may attain attributable to buoyancy and momentum. A plume ascending and dispersing in neutral or unstable conditions will continue to expand vertically because of turbulent diffusion (eddies) after its momentum and buoyancy are spent. A number of procedures have been used by other investigators to establish the plume height resulting from plume buoyancy and momentum. In this study a relatively simple procedure, considered to be quite realistic, was evolved. This procedure (Carpenter, Frizzola, Smith, Leavitt & Thomas 1968) was based on preliminary results of limited analyses of these data. Plume rise, Δh, was defined as elevation of the plume at the point in distance and space where rise of the plume centreline as a function of distance reached a minimum value or became constant. This is a finite value which can be derived from the observational data. The plume often continued to ascend beyond this point. With this criterion, effective termination of plume rise normally occurred 457 to 1219 m from the source. For uniformity, the Δh values used for comparison with calculated values throughout this study were the observed values at 1.2 km. In most instances observations extended well beyond this point.

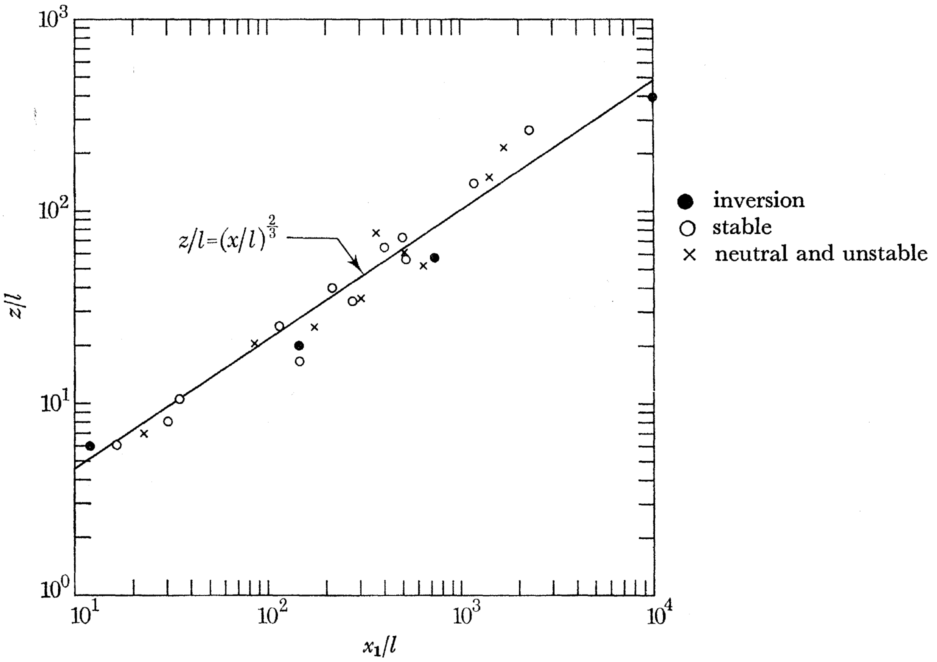

FIGURE 2. End of initial plume rise phase.

In addition, to a definition of criteria for establishing plume rise, it is desirable to determine whether one is observing the initial phase where source effects and mean winds are important. Difference in semantics and interests on these questions probably accounts for a large portion of the differences in plume rise estimates. The data may be examined in respect to the initial plume rise phase in accordance with the relation set out by Csanady (1961) who defines the initial flux, F, due to buoyancy and momentum as

$$F = gV_s r^2 \Delta T/T. \tag{2}$$

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TABLE 3. x/l AND z/l VALUES—INITIAL PLUME RISE PHASE

Paradise steam plant

1†		1		2		2		2		2		2	
x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l
43.1	14.7	245.8	51.6	2.1	3.3	6.8	4.8	24.6	14.8	29.3	12.3	73.3	21.1
86.1	21.5	491.6	96.8	4.3	4.7	13.6	8.0	49.2	22.9	58.6	19.6	146.5	34.6
172.2	31.6	983.3	154.8	8.5	6.7	27.2	12.4	98.3	32.9	117.2	26.9	293.1	53.8
258.3	36.2	1475.0	200.0	12.8	7.7	40.8	15.5	147.5	40.6	175.8	33.1	439.6	69.2
344.4	41.2	1967.0	248.4	17.1	8.4	54.4	17.4	196.7	47.4	234.5	42.3	586.2	82.7
430.5	45.8	2458.0	293.6	21.3	9.2	68.0	19.5	245.8	52.3	293.1	51.5	732.7	95.2
516.6	48.6	2950.0	335.5	25.6	9.6	81.7	21.1	295.0	57.4	351.7	58.9	879.6	102.9
602.7	53.7	3441.0	367.8	29.9	10.1	45.3	23.6	344.1	61.6	410.3	64.2	1026.0	127.0
688.8	57.6	3933.0	400.0	34.2	10.4	108.9	24.9	393.3	65.5	468.9	59.2	1172.0	144.2

2		3		3		3		3		3	
x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l
143.8	34.0	1.4	1.7	10.5	8.1	29.2	10.3	39.5	13.5	100.3	32.9
287.6	64.2	2.7	2.9	21.0	13.0	58.4	16.9	79.0	21.8	200.6	57.9
575.2	103.8	5.5	4.1	42.0	15.6	116.7	24.9	157.9	30.1	401.5	85.5
862.7	139.6	8.2	4.8	63.0	16.7	175.1	31.8	236.9	33.2	601.7	110.5
1150.3	179.3	10.9	5.4	84.1	20.8	233.5	40.6	315.8	40.9	802.2	147.4
1438.0	211.3	13.7	5.9	105.1	23.2	291.9	48.3	309.8	50.8	1003.0	151.3
1726.2	235.9	16.4	6.1	126.2	26.3	350.5	55.2	474.0	53.4	1203.9	175.0
2013.1	262.3	19.1	6.3	147.1	29.5	408.7	63.2	552.7	56.5	1404.0	202.7
2300.0	283.1	21.8	6.7	168.1	24.6	467.1	72.4	631.7	50.8	1604.5	232.9

† Stability classification data group.

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TABLE 3 (*cont.*)

Gallatin steam plant

1†		2		2		2		2		3		3		3	
x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l	x/l	z/l
8.8	5.3	1.8	2.7	12.9	5.9	30.5	6.4	94.1	32.1	5.8	4.7	24.6	10.0	87.6	27.6
17.6	7.7	3.6	3.3	25.8	9.8	61.0	12.4	188.2	56.8	11.5	6.4	49.2	16.8	175.2	51.7
35.1	12.6	7.2	3.9	51.7	16.3	121.9	24.8	376.4	88.9	23.1	9.0	98.3	30.3	350.3	72.4
52.7	16.4	10.8	5.1	77.5	22.4	182.9	37.2	564.6	112.3	34.6	11.4	147.5	43.6	525.3	93.1
70.2	18.0	14.4	5.9	103.3	26.8	243.8	49.6	752.9	127.2	46.2	13.2	196.7	56.1	700.4	103.4
87.8	18.8	18.0	6.6	129.2	30.8	304.8	62.0	941.1	134.6	57.7	15.6	245.8	64.5	875.5	112.6
105.4	19.1	21.6	6.8	155.1	34.6	365.9	66.8	1130.0	176.6	69.3	17.3	295.1	72.9	1051.1	125.2
122.9	19.4	25.2	7.5	180.8	37.1	426.7	69.2	1317.5	203.8	80.8	19.0	234.1	79.7	1226.0	125.2
140.5	19.7	28.8	7.9	206.7	39.3	487.7	72.0			92.4	19.9			1401.0	157.4.

Widows Creek steam plant

1*		2		2		2		3	
x/l	z/l	x/l	z/l	x/l	z/l	z/l	z/l	x/l	z/l
1.2	3.1	1.7	3.5	8.2	6.4	22.2	6.1	18.0	5.7
2.5	4.2	3.3	4.4	16.3	9.1	44.4	12.2	35.9	13.7
4.9	5.2	6.6	5.4	32.7	12.9	88.9	17.8	71.9	16.0
7.3	5.7	9.9	6.2	49.0	15.1	133.3	22.4	107.8	18.2
9.8	5.5	13.2	6.7	65.3	17.0	177.7	27.1	143.7	23.3
12.2	6.1	16.5	6.9	81.7	19.0	222.1	30.0	179.7	24.3
				98.1	24.4	266.7	34.1	215.7	29.5
				114.4	16.0	311.0	39.7	251.6	28.1

† Stability classification data group.

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Table 4. Principal operational and meteorological parameters

steam plant number	number of observations	V_s m s^{-1}	$\bar{u}_4$ m s^{-1}	$\bar{u}_3$ m s^{-1}	$\bar{u}_2$ m s^{-1}	$\bar{u}_1$ m s^{-1}	$\bar{u}$ m s^{-1}	$\Delta\theta/\Delta z$ K/100 m	ΔT K	T K	$10^{-7}Q_H$ cal s^{-1}	$10^{-2}Q$ m^3 s^{-1}	Δh m
1	1	15.6	4.8	6.4	12.4	9.4	8.5	1.73	120	291	1.70	5.46	89
	2	—	5.5	4.6	7.6	6.0	6.8	-0.05	113	298	1.70	5.59	179
	3	—	4.7	4.6	5.5	5.0	4.9	-0.09	111	300	1.70	5.63	256
	4	16.4	7.9	8.8	9.3	9.2	8.1	1.37	121	291	1.86	5.70	115
	5	—	4.8	6.0	7.3	6.7	6.0	0.85	120	293	1.86	5.73	188
	6	—	2.3	2.2	3.4	2.8	2.6	0.00	116	297	1.86	5.81	375
	7	—	5.3	5.0	3.9	4.5	4.7	0.01	114	298	1.86	5.84	195
	8	17.8	5.4	4.4	2.6	3.5	4.1	0.44	119	293	2.02	6.25	295
	9	—	2.7	2.9	3.5	3.2	3.3	0.17	117	295	2.02	6.30	363
	10	—	3.5	1.5	2.5	2.0	1.9	0.01	113	299	2.02	6.38	331
	11	17.1	4.3	3.5	4.7	4.1	4.5	0.59	119	293	1.87	6.02	329
	12	—	2.7	3.7	4.2	3.9	3.7	0.65	120	292	1.87	5.99	363
	13	—	1.6	2.7	5.2	3.9	3.4	0.30	116	295	1.87	6.06	457
	21	15.3	5.1	7.6	8.7	8.1	7.3	-0.17	120	297	1.78	5.36	242
	22	15.6	4.8	6.0	9.2	7.5	7.7	0.28	124	292	1.84	5.40	219
	23	—	9.9	8.8	11.6	10.3	10.9	-0.04	121	294	1.84	5.44	177
	24	—	6.9	7.7	7.0	7.3	7.2	-0.13	118	297	1.84	5.50	145
	25	—	11.3	10.9	10.0	10.5	11.4	0.18	125	291	1.84	5.37	135
	26	18.0	10.3	10.4	9.8	10.2	10.8	0.01	137	275	2.18	5.93	110
	27	—	10.2	10.0	9.8	10.0	9.5	0.00	137	275	2.18	5.94	108
	28	—	9.8	7.3	10.1	8.8	10.0	0.02	136	277	2.18	5.97	94
	29	—	7.4	8.0	7.3	7.6	8.3	-0.02	136	277	2.18	5.97	109
	30	16.4	6.9	4.5	5.1	4.8	5.2	0.08	137	279	2.00	5.44	308
	31	—	5.9	5.8	5.0	5.4	5.7	-0.02	135	281	2.00	5.48	178
	38	19.1	10.7	13.3	21.9	17.9	16.8	1.12	140	283	2.44	6.30	124
	39	—	9.6	11.3	19.1	15.3	13.8	0.37	137	286	2.44	6.35	150
	40	—	8.2	8.2	14.2	10.7	10.6	0.02	135	288	2.44	6.40	165
	41	—	8.2	7.5	8.2	7.8	8.4	0.09	132	291	2.44	6.47	154
	42	19.2	5.2	5.7	9.1	7.3	8.0	0.27	145	279	2.46	6.23	191
	43	—	4.6	4.6	6.8	5.8	5.2	0.11	144	280	2.46	6.25	182
2	3	15.4	3.7	6.1	1.8	4.0	2.8	0.63	111	299	1.76	5.11	312
	4	—	2.9	2.6	2.2	2.4	2.5	0.41	114	296	1.76	5.07	303
	6	16.1	3.8	4.9	8.2	6.5	5.9	0.72	116	294	1.74	5.26	165
	7	—	4.8	3.2	6.6	4.9	5.0	0.22	114	296	1.74	5.29	207
	8	—	5.2	5.8	4.8	5.3	5.8	0.11	111	299	1.74	5.34	215
	9	16.4	2.5	5.2	8.6	6.8	5.4	0.39	113	297	1.75	5.41	208
	10	15.4	5.8	6.8	9.4	8.2	6.9	0.30	112	298	1.73	5.11	180
	11	—	6.5	6.4	7.4	7.0	6.9	-0.16	109	301	1.73	5.16	130
	12	15.3	3.8	2.9	4.1	3.5	3.8	-0.03	110	300	1.70	5.09	214
	13	—	2.4	1.8	2.1	1.9	2.2	0.09	109	301	1.70	5.11	460
	14	—	2.6	2.2	1.7	1.8	2.1	0.23	110	300	1.70	5.09	555
	15	—	2.8	—	1.4	—	2.1	0.30	110	300	1.70	5.10	476
	16	15.6	2.8	4.6	6.5	5.5	5.1	0.74	114	296	1.71	5.15	226
	17	—	1.8	3.8	3.7	3.7	2.9	0.44	112	298	1.71	5.18	317
	18	—	5.3	4.6	8.7	6.7	6.1	-0.02	107	203	1.71	5.26	314
	19	15.5	3.3	3.6	8.6	6.2	5.0	0.68	113	297	1.65	5.13	220
	20	—	4.6	5.2	6.1	5.7	5.8	-0.04	106	304	1.65	5.25	354
	21	15.3	1.7	4.8	4.3	4.6	4.6	1.06	115	295	1.77	5.03	171
	22	—	1.6	4.2	3.0	3.6	3.2	0.97	113	297	1.77	5.06	251
	23	—	2.7	2.6	1.9	2.3	2.5	0.36	111	299	1.77	5.09	347
	24	14.8	2.8	4.0	5.3	4.7	5.2	0.61	124	282	1.50	4.67	359
	25	—	4.1	6.0	3.6	4.8	4.6	0.10	121	286	1.50	4.73	259
	26	—	8.2	7.9	11.3	9.8	9.9	0.00	117	289	1.50	4.78	137
	27	—	6.7	11.6	11.7	11.7	10.4	0.13	117	290	1.50	4.79	165
	28	15.2	2.7	3.3	5.1	4.3	3.1	0.06	133	273	1.52	4.66	250
	29	—	4.8	4.2	5.7	4.9	4.4	0.00	131	275	1.52	4.69	263
6	1	22.9	4.7	5.1	5.7	5.4	6.5	0.19	119	283	1.77	5.08	136
	2	—	6.6	6.4	5.3	5.8	6.0	-0.17	116	286	1.77	5.14	125
	3	—	6.0	6.8	4.1	5.5	4.8	0.02	115	287	1.77	5.16	155
	4	—	1.5	3.3	4.3	3.8	2.5	1.32	121	279	1.79	5.04	377
	5	—	1.9	3.5	4.1	3.8	3.4	0.49	117	283	1.79	5.12	304
	6	—	3.5	1.6	2.2	1.8	2.2	0.04	110	290	1.79	5.24	333
	7	24.5	5.9	6.0	6.2	6.1	6.2	-0.18	118	280	1.59	5.44	73
	8	22.9	5.7	6.1	4.8	5.5	5.2	0.22	125	275	1.57	4.97	138
	9	—	3.3	3.2	5.7	4.4	4.1	0.25	122	278	1.57	5.03	301
	10	—	2.0	2.9	2.7	2.8	2.4	0.09	118	281	1.57	5.09	316

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Dividing this flux by the cube of the horizontal wind speed, $\bar{u}^3$, obtains a length,

$$l = F/\bar{u}^3. \tag{3}$$

The distance variables divided by l, i.e. z/l and x/l, are dimensionless and

$$z/l = f(x/l), \tag{4}$$

where z is plume rise and x is distance downwind. At the point x_1/l where the plume levels off, z/l becomes constant. Values of z/l and x/l were calculated for the principal wind speed and stability groupings at the nine distances out to 1.219 km. In most instances, a levelling of z/l values is evident as x/l approaches the 1.219 km distance. The values of the distance variables where z/l becomes constant for the twenty-six summary observations in table 3 are plotted in figure 2. Here a two-thirds slope, as observed by Csanady (1961) for the '$\frac{2}{3}$ power law' relation, fits the data quite well

$$z/l = (x/l)^{\frac{2}{3}}. \tag{5}$$

Compilation—estimates of quantities for plume rise calculations

Either direct measurements of reliable estimates of every quantity needed for computation of plume rise according to principal formulas now in use were compiled for each of the composite observations along with observed plume rise. Values of these parametres for the sixty-six composite observations based on single stack operations are shown in table 4.

Comparison between observed and calculated Δh values from major plume rise formulas

The observed plume rise was examined for correlation with principal existing plume rise formulas. Formulas selected for comparison were Holland (1953); Bosanquet, Carey & Halton (1950); Davidson-Bryant (1949); Csanady (1961); Concawe (Brummage *et al.* 1966); and Lucas, Moore & Spurr (1963). The formula was applied in each case as recommended by the author without benefit of adjustment factors suggested by later investigators. Plume rise was

TABLE 5. OBSERVED AND CALCULATED PLUME RISE—PRINCIPAL FORMULAS

					calculated plume rise/m					
steam plant number	number of obser-vations	stability class	wind speed/ m s^{-1}	observed plume rise/m	Holland	Bosanquet, Carey & Halton	Davidson-Bryant	Csanady	Concawe	Lucas, Moore & Spurr
1	4	1	8.1	115	116	142	27	310	157	147
	1	1	8.5	89	102	121	24	245	145	132
	38	1	16.8	124	72	47	13	59	104	115
	10	2	1.9	331	536	6000	231	23382	486	1058
	9	2	3.3	363	309	1061	107	4768	321	395
	13	2	3.4	457	280	831	97	4471	302	332
	12	2	3.7	363	257	589	87	3656	284	262
	8	2	4.1	295	249	580	80	2581	273	280
	11	2	4.5	329	211	456	66	1998	245	243
	7	2	4.7	195	200	898	58	1401	236	659
	30	2	5.2	308	191	602	52	1661	227	379
	43	2	5.2	182	233	643	66	2241	252	369
	5	2	6.0	188	156	257	42	734	197	192
	22	2	7.7	219	120	198	28	348	162	223
	42	2	8.0	191	151	237	36	624	183	238
	41	2	8.4	154	143	234	33	423	175	305
	28	2	10.0	94	109	188	24	284	145	395
	40	2	10.6	165	113	170	24	218	147	395
	26	2	10.8	110	100	170	22	230	137	452
	25	2	11.4	135	81	95	16	109	121	205
	39	2	13.8	150	87	75	16	102	121	167

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TABLE 5 (*cont.*)

steam plant number	number of obser-vations	stability class	wind speed/ m s^{-1}	observed plume rise/m	calculated plume rise/m: Holland	Bosanquet, Carey & Halton	Davidson-Bryant	Csanady	Concawe	Lucas, Moore & Spurr
	6	3	2.6	375	361	2256	133	8434	369	584
	3	3	4.9	256	177	111	51	1081	219	303
	31	3	5.7	178	174	381	46	1233	212	271
	2	3	6.8	179	127	194	32	451	171	218
	24	3	7.2	145	128	273	30	391	171	210
	21	3	7.3	242	122	167	29	393	166	206
	29	3	8.3	109	131	144	31	497	167	190
	27	3	9.5	108	114	103	26	338	151	166
	23	3	10.9	177	84	18	17	118	125	139
2	21	1	4.6	171	192	331	52	1207	234	205
	14	2	2.1	555	407	1503	155	10827	414	440
	15	2	2.1	476	407	1339	155	10827	414	412
	13	2	2.2	460	388	2033	145	9310	399	544
	4	2	2.5	303	352	1007	124	7358	369	352
	23	2	2.5	347	353	1037	122	6977	370	364
	3	2	2.8	312	314	730	105	4902	339	299
	17	2	2.9	317	297	814	102	4393	326	319
	28	2	3.1	250	252	1573	93	6509	292	493
	22	2	3.2	251	276	540	87	3473	308	251
	25	2	4.6	259	167	598	51	1319	216	355
	7	2	5.0	207	176	430	50	1007	218	290
	19	2	5.0	220	167	324	47	905	213	216
	16	2	5.1	226	169	311	46	845	213	211
	24	2	5.2	359	148	327	43	977	197	212
	9	2	5.4	208	164	333	46	826	207	242
	8	2	5.8	215	164	360	40	605	195	320
	6	2	5.9	165	149	253	40	632	193	199
	10	2	6.9	180	126	212	30	368	171	228
	27	2	10.4	165	74	103	16	105	117	221
	12	3	3.8	214	225	600	68	1827	265	391
	29	3	4.4	263	178	658	57	2039	225	328
	20	3	5.8	354	144	252	38	500	190	254
	18	3	6.1	314	141	225	36	424	186	244
	11	3	6.9	130	126	156	30	349	171	216
	26	3	9.9	137	78	69	17	125	121	145
6	4	1	2.5	377	373	565	182	11471	372	264
	6	2	2.2	333	424	2492	213	14186	410	675
	10	2	2.4	316	352	1772	192	13734	360	510
	5	2	3.4	304	274	614	118	4304	296	290
	9	2	4.1	301	206	595	91	2864	241	303
	3	2	4.8	155	196	706	78	1495	227	542
	8	2	5.2	138	162	418	66	1458	201	277
	1	2	6.5	136	142	274	48	638	181	265
	2	3	6.0	125	154	217	53	778	192	250

calculated with these equations (table 5), and plotted against observed values for single stack operations (figures 3 to 8). Points in these plots are coded according to wind speeds less than 3 m s^{-1} and equal to or greater than 3 m s^{-1}.

The relation of observed plume rise with calculated plume rise shown by the various formulas is as follows:

(1) Holland formula (figure 3) shows fairly good agreement with a tendency to slightly underestimate plume rise.

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(2) Bosanquet *et al.* formula (figure 4) overestimates plume rise and induces large scatter.

(3) Davidson–Bryant formula (figure 5) seriously underestimates plume rise.

(4) Csanady formula (figure 6) seriously overestimates plume rise, but with moderate scatter.

(5) Concawe formula (figure 7) shows good aggreement.

(6) Lucas *et al.* formula (figure 8) has a tendency to overestimate the plume rise and induce moderate scatter.

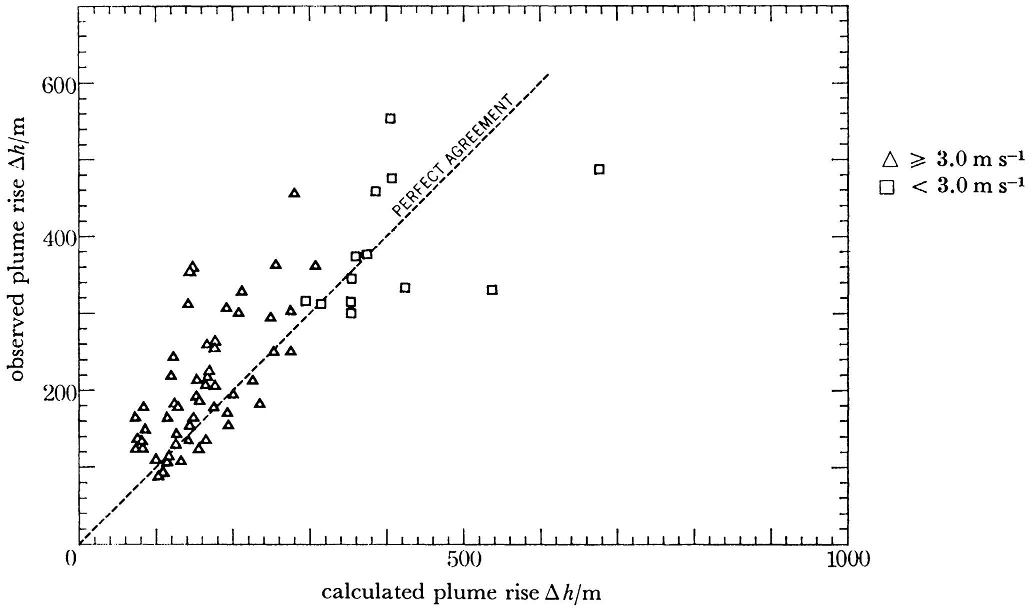

FIGURE 3. Relation between observed plume rise and that calculated by the Holland formula:

$$\Delta h = (1.5\,V_{\mathrm{s}}d + 4\times 10^{-5}Q_{\mathrm{H}}/\bar{u}.$$

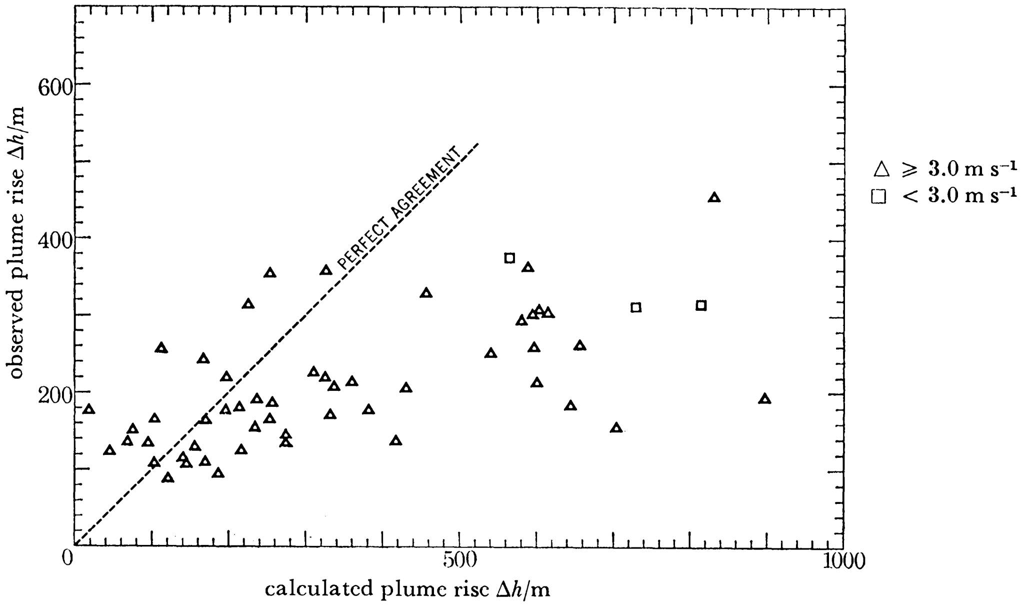

FIGURE 4. Relation between observed plume rise and that calculated by the Bosanquet, Carey & Halton formula:

$$\Delta h = \frac{4.77(QV_{\mathrm{s}})^{\frac{1}{2}}}{(1+0.43\bar{u}/V_{\mathrm{s}})\,\bar{u}} + \frac{6.47gQD(\ln J^2+2/J-2)}{\bar{u}^3T}$$

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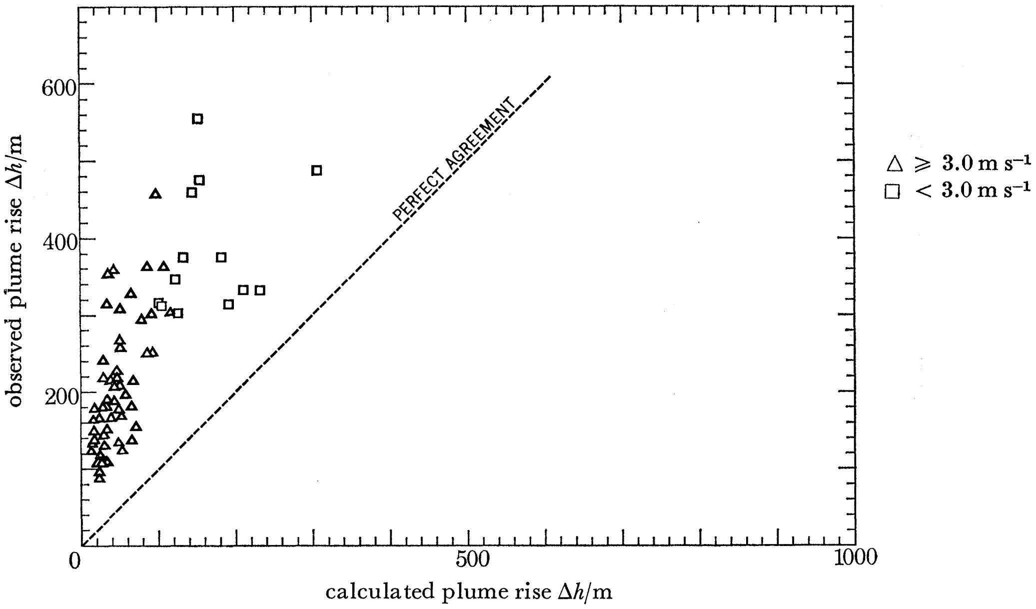

Figure 5. Relation between observed plume rise and that calculated by the Davidson-Bryant formula: $\Delta h = d(V_s/\bar{u})^{1.4}\,(1+\Delta T/T_s)$

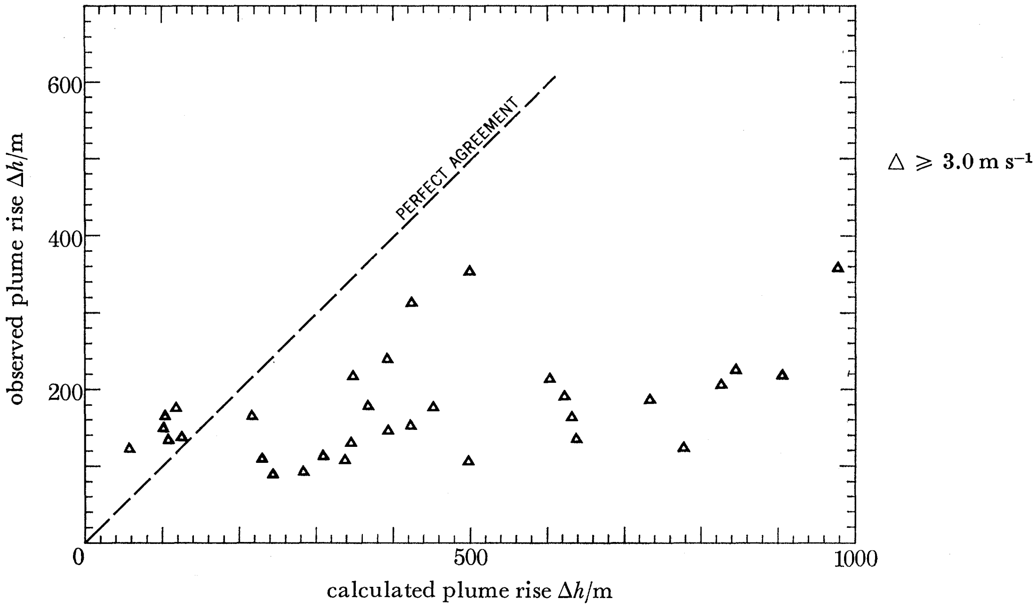

Figure 6. Relation between observed plume rise and that calculated by the Csanady formula: $\Delta h = 250\,F/\bar{u}^3$.

Individual points shown in these plots represent specific observations averaged over periods when some variability in meteorological parameters was inevitable and some fluctuation would normally be expected in the results.

Optimization of formulas based on observed plume rise

On the basis of relation shown in table 5 where observed and calculated plume rise values are listed and in figures 3 to 8 where these values are plotted, the Concawe and Csanady plume rise

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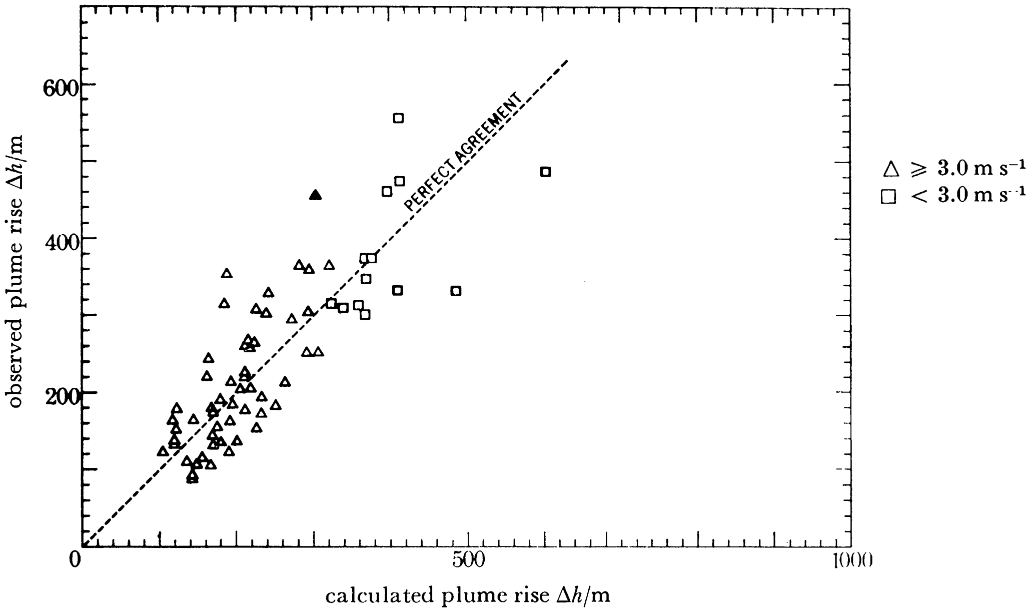

FIGURE 7. Relation between observed plume rise and that calculated by the Concawe formula: $\Delta h = 0.175\, Q_H^{\frac{1}{2}}/\bar{u}^{\frac{3}{4}}$.

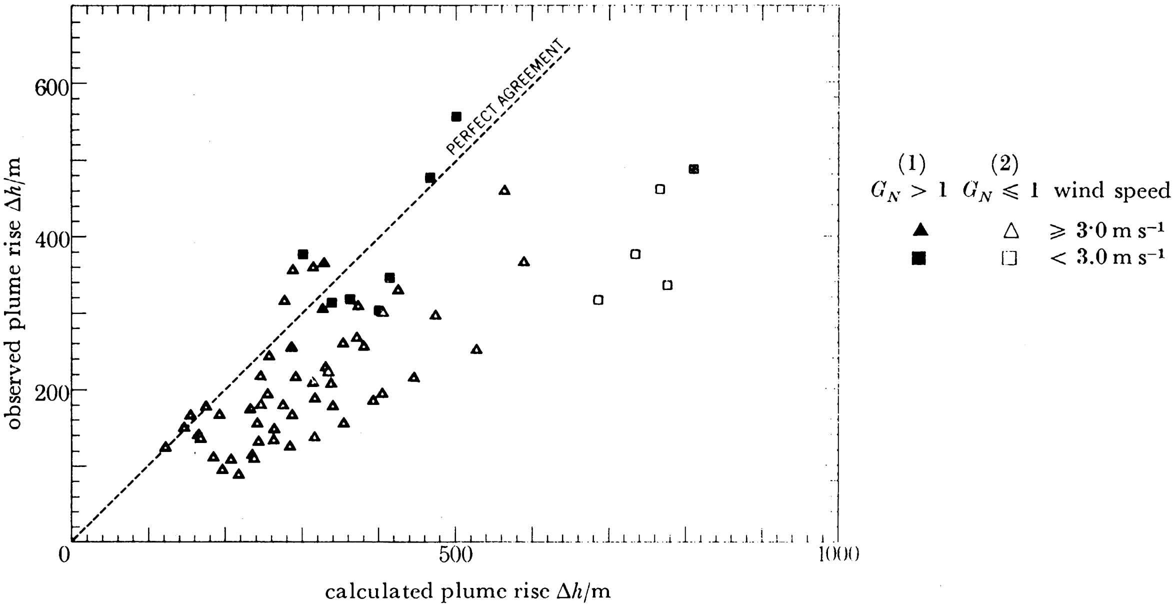

FIGURE 8. Relation between observed plume rise and that calculated by the Lucas, Moore & Spurr formulae: (1) $\Delta h = (0.7^{\alpha}/\bar{u})\,(Q_N/G_N)^{\frac{1}{4}}$ and (2) $\Delta h = \alpha Q_N^{\frac{1}{4}}/\bar{u}$, where $\alpha = 475 + 2(h_c - 100)$ m² s⁻¹ MW$^{-\frac{1}{4}}$.

formulas were selected for optimization by the process of multiple regression for best conformance with the plume rise values observed in this extensive study. In this process the basic elements of each formula were retained, but coefficients and exponents were modified to yield best agreement with observations. Fit of the basic formula with observed values, amenability to the regression technique, and inclusion of significant meteorological parameters were the bases of this selection. In optimizing these formulas, only observations where analyses indicated that full rise attributable

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Table 6. Observed and calculated plume rise optimized and two-thirds power law formulas

steam plant number	number of observations	stability class	wind speed/ m s^{-1}	observed plume rise/m	calculated plume rise/m		
					Concawe (optimized)	Csanady (optimized)	$\frac{2}{3}$ power law
1	4	1	8.1	115	164	141	140
	1	1	8.5	89	152	128	128
	38	1	16.8	124	111	88	76
	10	2	1.9	331	465	523	645
	9	2	3.3	363	317	333	374
	13	2	3.4	457	300	317	354
	12	2	2.7	363	283	289	323
	8	2	4.1	295	273	274	298
	11	2	4.5	329	247	248	266
	7	2	4.7	195	239	246	255
	30	2	5.2	308	230	241	249
	43	2	5.2	182	252	254	266
	5	2	6.0	188	202	189	194
	22	2	7.7	219	169	163	156
	42	2	8.0	191	187	177	172
	41	2	8.4	154	180	167	158
	28	2	10.0	94	152	146	134
	40	2	10.6	165	153	140	127
	26	2	10.8	110	144	138	125
	25	2	11.4	135	129	120	106
	39	2	13.8	150	128	110	96
	6	3	2.6	375	360	400	464
	3	3	4.9	256	223	235	239
	31	3	5.7	178	216	225	227
	2	3	6.8	179	178	181	173
	24	3	7.2	145	177	176	167
	21	3	7.3	242	173	175	165
	29	3	8.3	109	173	171	162
	27	3	9.5	108	157	154	142
	23	3	10.9	177	133	126	111
2	21	1	4.6	171	237	218	234
	1	2	1.0	1 025	682	800	1 114
	2	2	1.3	488	569	657	859
	5	2	1.3	777	569	649	859
	14	2	2.1	555	402	440	528
	15	2	2.1	476	402	437	526
	13	2	2.2	460	389	428	506
	4	2	2.5	303	361	381	447
	23	2	2.5	347	362	378	442
	3	2	2.8	312	334	336	389
	17	2	2.9	317	322	336	383
	28	2	3.1	250	292	252	396
	22	2	3.2	251	305	293	335
	25	2	4.6	259	220	243	252
	7	2	5.0	207	222	224	230
	19	2	5.0	220	217	211	220
	16	2	5.1	226	218	208	216
	24	2	5.2	359	202	211	219
	9	2	5.4	208	211	207	211
	8	2	5.8	215	200	199	197
	6	2	5.9	165	198	188	190
	10	2	6.9	180	177	168	162
	27	2	10.4	165	125	124	110

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TABLE 6 (*cont.*)

steam plant number	number of observations	stability class	wind speed/m s^{-1}	observed plume rise/m	calculated plume rise/m		
					Concawe (optimized)	Csanady (optimized)	$\frac{2}{3}$ power law
2	12	3	3.8	214	267	279	305
	29	3	4.4	263	229	265	278
	20	3	5.8	354	196	196	192
	18	3	6.1	314	192	189	183
	11	3	6.9	130	177	174	164
	26	3	9.9	137	130	130	116
6	4	1	2.5	377	364	360	443
	6	2	2.2	333	398	439	520
	10	2	2.4	316	353	418	492
	5	2	3.4	304	294	302	337
	9	2	4.1	301	247	270	290
	3	2	4.8	155	230	237	243
	8	2	5.2	138	207	226	231
	1	2	6.5	136	187	185	181
	2	3	6.0	125	197	202	198

to momentum and buoyancy had been attained were considered; thus the optimized calculations shown in table 6 were based on the twenty-six summary observations according to wind speed and stability classification.

The Csanady formula with the $\bar{u}^3$ term in the denominator is obviously not useful with low or high wind speeds. However, it was selected for optimization because it contains the significant meteorological parameters required for evaluating the effect of ambient conditions, especially stability. The simple Concawe formula was selected because of its superior agreement with observations. It is considered that optimization of these formulas over the broad range of plant designs, operational factors, and meteorological conditions encompassed in this study should either confirm the efficacy of the formula as initially presented or result in some improvement for application to large power plants.

When programmed for optimization the original Csanady formula

$$\Delta h = 250F/\bar{u}^3 \tag{6}$$

became

$$\Delta h = 133\,\mathrm{m}^{0.73}(F/\bar{u}^3)^{0.72} \tag{7}$$

for the full range of stability conditions.

Data falling into each of the three stability classes were then reprogrammed by means of the following formula:

$$\Delta h = C_1(F/\bar{u}^3)^{0.27}, \tag{8}$$

from which the following values of C_1 were determined:

Stability class 1 (0.013 K m^{-1}, average potential temperature gradient); $C_1 = 119$.

Stability class 2 (0.003 K m^{-1}, average potential temperature gradient); $C_1 = 131$.

Stability class 3 (-0.0006 K m^{-1}, average potential temperature gradient); $C_1 = 137$.

A linear variation of C_1 with temperature gradient permits interpolation of C_1 values for intermediate gradient values.

By using the final formula with the appropriate C_1 values, the values of Δh were calculated and are plotted against observations in figure 9. The optimized formula reduced the scatter considerably from that shown in figure 6 based on the original formula.

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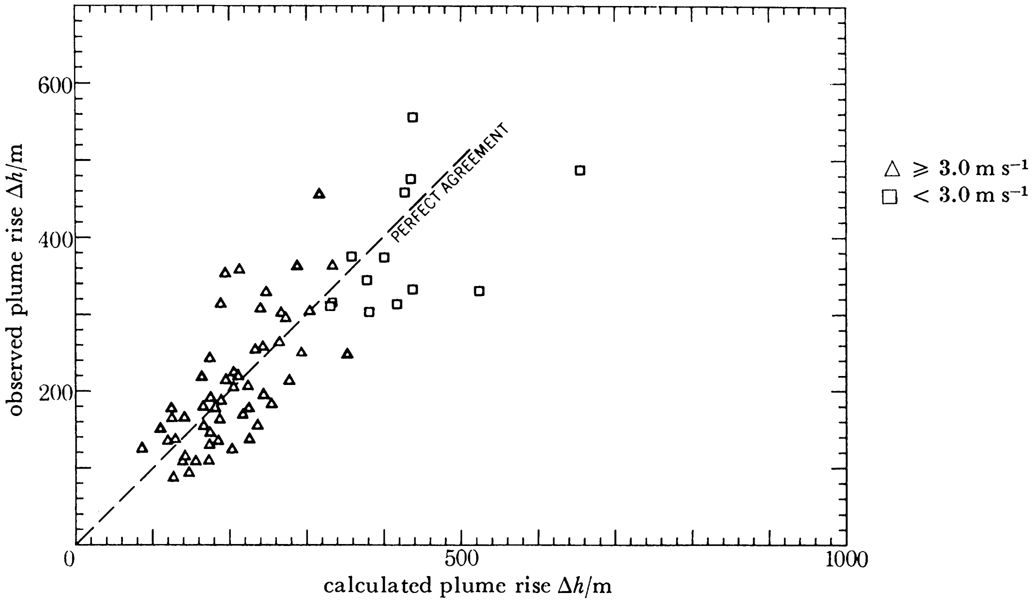

Figure 9. Relation between observed plume rise and that calculated by the optimized Csanady formula: $\Delta h = C_1(F/\bar{u}^3)^{0.27}$.

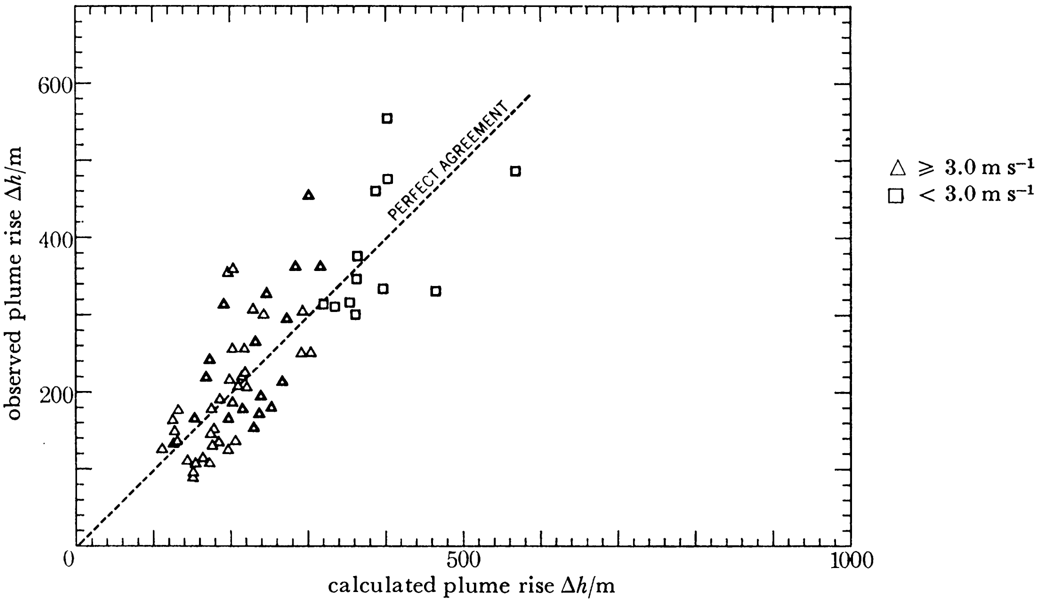

Figure 10. Relation between observed plume rise and that calculated by the optimized Concawe formula:

$$\Delta h = 0.414 \frac{Q_{\mathrm{H}}^{0.444}}{\bar{u}^{0.694}} \Bigg/ \frac{\mathrm{m}^{1.694}}{\mathrm{cal}^{0.444}\,\mathrm{s}^{0.250}}.$$

The original Concawe formula derived by multiple regression from observations of Raugh (1964) in Europe showed very good agreement with observations at T.V.A. steam plants, figure 10. When this formula

$$\Delta h = 0.175[Q_{\mathrm{H}}^{\frac{1}{2}}/\bar{u}^{\frac{3}{4}}] \tag{9}$$

was optimized for best conformity with T.V.A. data, it took the form

$$\Delta h = 0.414[Q_{\mathrm{H}}^{0.444}/\bar{u}^{0.694}]. \tag{10}$$

This optimization resulted in slightly less scatter.

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‘Two-thirds power law’ relation

Returning to the relation shown between the distance variables in figure 2, the ‘two-thirds power law’ plume rise formula can be derived as

$$\Delta h = CF^{\frac{1}{3}}u^{-1}x^{\frac{2}{3}}. \tag{11}$$

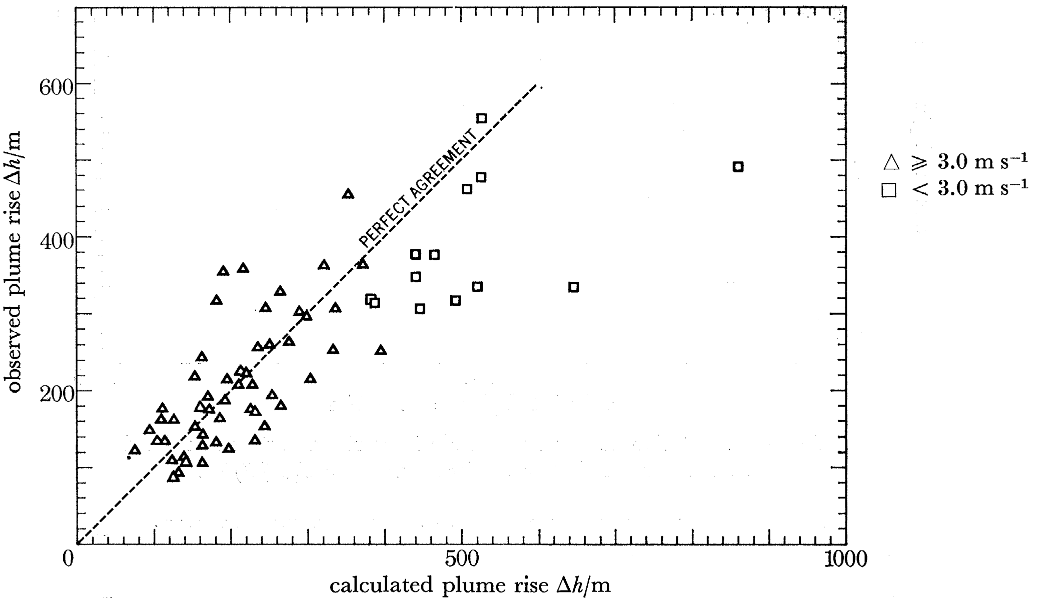


Figure 11. Relation between observed plume rise and that calculated by the ‘⅔ power law’ relation: $\Delta h = C(114\ \mathrm{m}^{\frac{2}{3}})\ F^{\frac{1}{3}}/\bar{u}$.

The observed Δh values at $x = 1.219$ km are then calculated (table 6 and figure 11), as

$$\Delta h = C(114\ \mathrm{m}^{\frac{2}{3}})\ F^{\frac{1}{3}}u^{-1}, \tag{12}$$

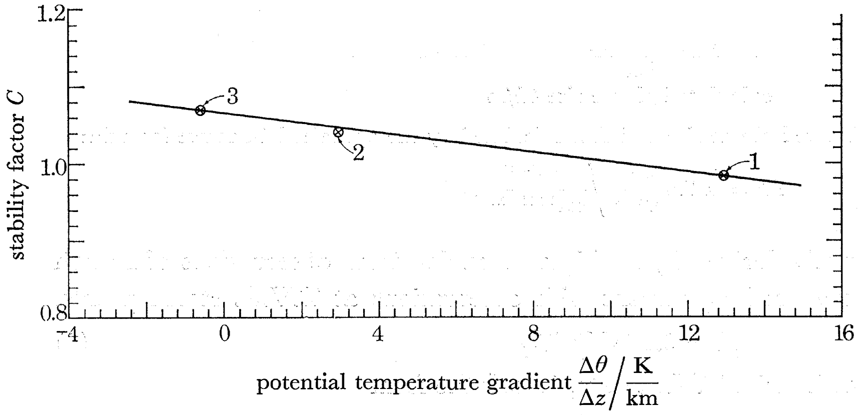
where C is a dimensionless stability coefficient for which the following values are determined:

Stability class 1 ($0.013\ \mathrm{K\,m^{-1}}$, average potential temperature gradient); $C = 1.07$.

Stability class 2 ($0.003\ \mathrm{K\,m^{-1}}$, average potential temperature gradient); $C = 1.04$.

Stability class 3 ($-0.0006\ \mathrm{K\,m^{-1}}$, average potential temperature gradient); $C = 0.98$.

These values of C plotted against the potential temperature gradient, figure 12, show a straight line relation, i.e. the coefficient C becomes larger as the potential temperature decreases.



stability class	$\frac{\Delta\theta}{\Delta z} / \frac{\mathrm{K}}{\mathrm{km}}$	C (dimensionless)
1	13	0.98
2	3	1.04
3	−0.6	1.07

Figure 12. Stability classification. $\Delta h = C(F^{\frac{1}{3}}/\bar{u})\ x^{\frac{2}{3}}$. When $x = 1219$ m, $x^{\frac{2}{3}} = 114\ \mathrm{m}^{\frac{2}{3}}$ and $\Delta h = C(114\ \mathrm{m}^{\frac{2}{3}})\ F^{\frac{1}{3}}/\bar{u}$. $C = 1.065 - (6.25\ \mathrm{m/K})\ \Delta\theta/\Delta z$.

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The groundline concentration reaches a maximum value at a critical wind speed (Brummage *et al.* 1966) given as

$$u_c = [(2a-1)\,A/h_s]^{1/a}, \tag{13}$$

where a = exponent of wind velocity, h_s = actual stack height, A = function of kinetic and thermal energy of the plume. Applying this critical wind velocity concept, the effective stack height, H_e, at the critical wind speed, u_c, is calculated as

$$H_e = \frac{2a}{2a-1}\,h_s, \tag{14}$$

when the value of a is 1:

$$H_e = 2h_s. \tag{15}$$

Conclusions

Plume rise data plotted against calculated values of all formulas used in this report indicated that the wind speed and heat emission rate are the principal determinants in calculating plume rise. The effect of the stack height on plume rise is inherent in the stability factor for this analysis. When plume observations were plotted against calculated values according to the formulas originally presented by the author, the simple Concawe formula provided the best fit. Of the two formulas optimized to give best fit with T.V.A. observations, both the Concawe and Csanady formulas were good.

The Concawe formula is considered preferable for general investigation because of simplicity and ease of calculation. But when a particular event, such as inversion breakup or limited mixing layer fumigation, is being analysed, use of the 'two-thirds power law' is considered preferable, provided information for the meteorological parameters is available. This relation embodies the principal physical quantities normally associated with plume rise and permits some accounting for up to 15 % difference in plume rise attributable to variation in atmospheric stability.

The study served to validate plume rise formulas which can be used effectively over a range of meteorological and operational conditions. Plotting of the observed and calculated values shows some scatter even for the two best formulas. However, the scatter is limited and is equally distributed about a line of best fit. We doubt that the scatter can be reduced unless wind speed profiles are taken at less than 30 min intervals and related to shorter observation periods, say 5 to 10 min. However, it is unlikely that even a reduction in scatter would result in any substantive change in the formulas that have been developed. The common agreement of the original Concawe formula derived by regression analysis from several hundred observations in western Europe with the T.V.A. observations is judged to lend strength to this simple formula.

Nomenclature

a	exponent for mean horizontal wind speed (dimensionless)
A	some function of kinetic and thermal energy ($\mathrm{m^{1+a}\,s^{-1}}$)
b	buoyant acceleration at top of stack $= g(\rho_a-\rho)/\rho$ ($\mathrm{m\,s^{-2}}$)
C	stability coefficient (dimensionless)
C_1	stability coefficient ($\mathrm{m^{0.73}}$)
d	stack exit diameter (m)
D	difference between ambient temperature and stack gas temperature at stack top (K)

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e	Napierian base = 2.71828 (dimensionless)
F	flux due to buoyancy and momentum $= gV_sr^2\Delta T/T$ or V_sr^2b ($m^4\,s^{-3}$)
g	acceleration due to gravity ($m\,s^{-2}$)
G	change of potential temperature with height from stack top to plume top ($K\,m^{-1}$)
G_N	stability parameter $= \dfrac{108\,\Delta\theta/\Delta z}{\bar{u}^2}$ (dimensionless)
h_s	height of stack (m)
Δh	rise of the plume above the stack top (m)
H_e	effective stack height (m)
J	$= \dfrac{\bar{u}^2}{(QV_s)^{\frac{1}{2}}}\left[0.43\left(\dfrac{T_1}{gG}\right)^{\frac{1}{2}} - 0.28\,\dfrac{V_sT_1}{gD}\right] + 1$ (dimensionless)
l	$= F/\bar{u}^3$ (m)
Q	stack gas emission rate converted to temperature T_1 ($m^3\,s^{-1}$)
Q_H	heat emission ($cal\,s^{-1}$)
Q_N	heat emission (MW)
r	stack exit radius (m)
T	ambient air temperature (K)
T_a	ambient air temperature (K)
T_s	absolute temperature of stack gas (K)
T_1	temperature at which density of flue gases is equal to that of the atmosphere (K)
ΔT	temperature difference between exit stack gas and ambient air (K)
$\bar{u}_4$	mean horizontal wind speed at stack top ($m\,s^{-1}$)
$\bar{u}_3$	mean horizontal wind speed at plume bottom ($m\,s^{-1}$)
$\bar{u}_2$	mean horizontal wind speed at plume top ($m\,s^{-1}$)
$\bar{u}_1$	mean horizontal wind speed at plume centreline ($m\,s^{-1}$)
$\bar{u}$	mean horizontal wind speed between stack top and plume top ($m\,s^{-1}$)
u_c	critical wind speed ($m\,s^{-1}$)
V_s	stack gas exit velocity ($m\,s^{-1}$)
x	distance downwind from stack (m)
x_1	distance downwind from stack where the plume levels off (m)
z	predicted rise of the plume above the stack top for a given x (m)
α	stack height factor ($m^2\,s^{-1}\,MW^{-\frac{1}{4}}$)
$\Delta\theta/\Delta z$	change of potential temperature with height (K/100 m)
ρ	density of effluent ($g\,cm^{-3}$)
ρ_a	density of atmospheric air ($g\,cm^{-3}$)

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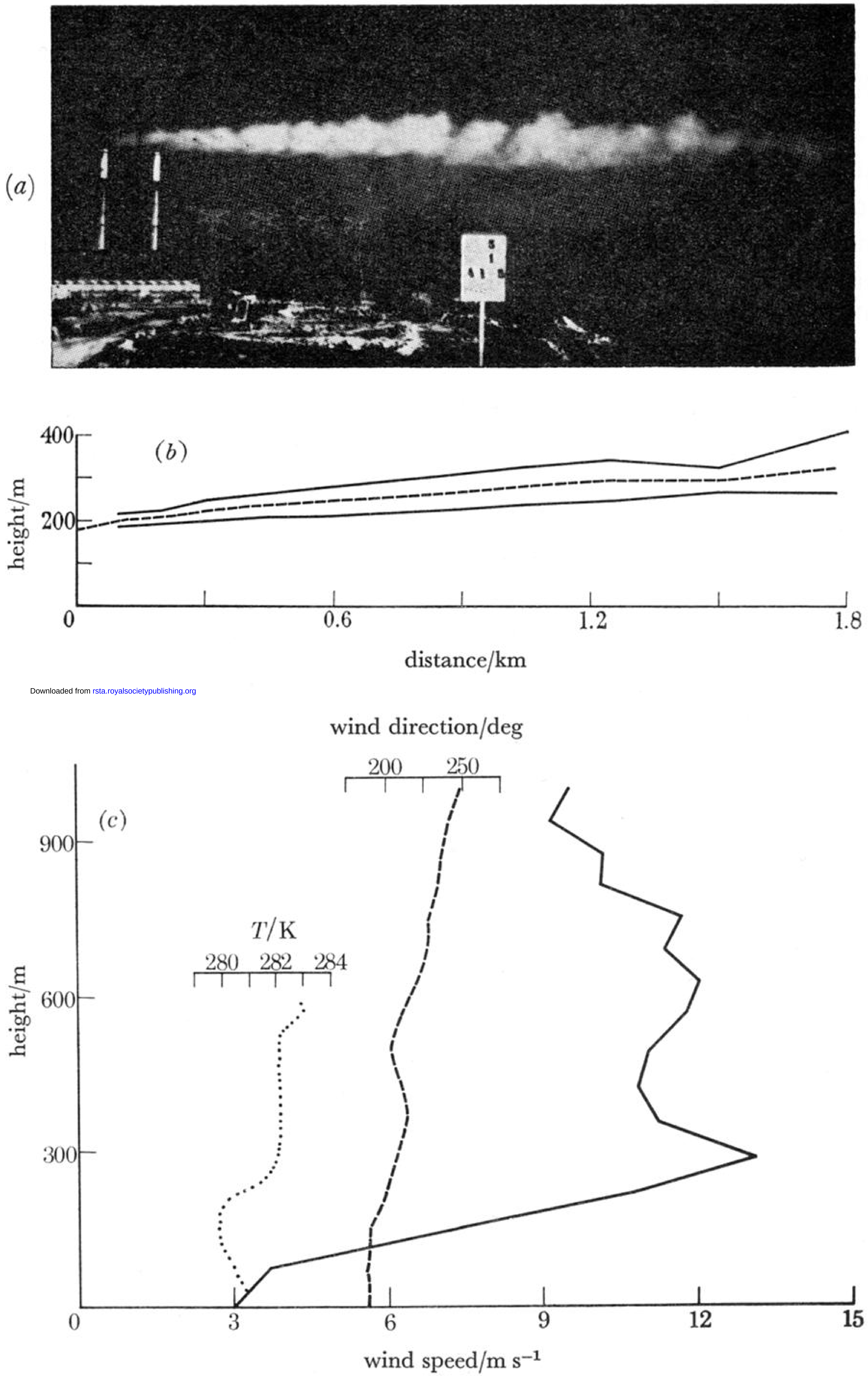

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IGURE 1. Data from typical day of field operation at the Paradise steam plant on 1 April 1965. (*a*) Plume photograph at 07.15 h; (*b*) wind speed and direction profile at 07.16 h, and temperature profile at 07.20 h; (*c*) plume profile derived from photographs: the stack height is 183 m.